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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

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Thursday 9 May 2013 (afternoon)

Examination code

2	2	1	3	–	7	2	0	5

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0116

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Find the exact value of $\int_1^2 \left((x-2)^2 + \frac{1}{x} + \sin \pi x \right) \mathrm{d}x$.

2. *[Maximum mark: 5]*

Consider the matrices $\boldsymbol{A} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}$.

(a) Find $\det \boldsymbol{A}$ and hence write down the matrix $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Find the matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$. *[3 marks]*

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3. *[Maximum mark: 4]*

Expand $(2-3x)^5$ in ascending powers of x, simplifying coefficients.

4. *[Maximum mark: 5]*

Tim and Caz buy a box of 16 chocolates of which 10 are milk and 6 are dark. Caz randomly takes a chocolate and eats it. Then Tim randomly takes a chocolate and eats it.

(a) Draw a tree diagram representing the possible outcomes, clearly labelling each branch with the correct probability. *[3 marks]*

(b) Find the probability that Tim and Caz eat the same type of chocolate. *[2 marks]*

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5. *[Maximum mark: 7]*

The curve C is given by $y = \frac{x\cos x}{x + \cos x}$, for $x \geq 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos^2 x - x^2 \sin x}{(x + \cos x)^2}$, $x \geq 0$. *[4 marks]*

(b) Find the equation of the tangent to C at the point $\left(\frac{\pi}{2}, 0\right)$. *[3 marks]*

6. *[Maximum mark: 7]*

A geometric sequence has first term a, common ratio r and sum to infinity 76. A second geometric sequence has first term a, common ratio r^3 and sum to infinity 36.

Find r.

7. *[Maximum mark: 7]*

Given the complex numbers $z_1 = 1 + 3\mathrm{i}$ and $z_2 = -1 - \mathrm{i}$.

(a) Write down the exact values of $|z_1|$ and $\arg(z_2)$. *[2 marks]*

(b) Find the minimum value of $|z_1 + \alpha z_2|$, where $\alpha \in \mathbb{R}$. *[5 marks]*

8. *[Maximum mark: 6]*

The curve C is given implicitly by the equation $\dfrac{x^2}{y} - 2x = \ln y$ for $y > 0$.

(a) Express $\dfrac{dy}{dx}$ in terms of x and y. *[4 marks]*

(b) Find the value of $\dfrac{dy}{dx}$ at the point on C where $y = 1$ and $x > 0$. *[2 marks]*

9. *[Maximum mark: 7]*

The function f is given by $f(x) = \dfrac{3^x + 1}{3^x - 3^{-x}}$, for $x > 0$.

(a) Show that $f(x) > 1$ for all $x > 0$. *[3 marks]*

(b) Solve the equation $f(x) = 4$. *[4 marks]*

10. *[Maximum mark: 6]*

(a) Given that $\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{8}\right)=\arctan\left(\frac{1}{p}\right)$, where $p \in \mathbb{Z}^{+}$, find p. *[3 marks]*

(b) Hence find the value of $\arctan\left(\frac{1}{2}\right)+\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{8}\right)$. *[3 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

The vertices of a triangle ABC have coordinates given by $A(-1, 2, 3)$, $B(4, 1, 1)$ and $C(3, -2, 2)$.

(a) (i) Find the lengths of the sides of the triangle.

(ii) Find $\cos B\hat{A}C$. *[6 marks]*

(b) (i) Show that $\overrightarrow{BC} \times \overrightarrow{CA} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$.

(ii) Hence, show that the area of the triangle ABC is $\frac{1}{2}\sqrt{314}$. *[5 marks]*

(c) Find the Cartesian equation of the plane containing the triangle ABC. *[3 marks]*

(d) Find a vector equation of (AB). *[2 marks]*

The point D on (AB) is such that $\overrightarrow{OD}$ is perpendicular to $\overrightarrow{BC}$ where O is the origin.

(e) (i) Find the coordinates of D.

(ii) Show that D does not lie between A and B. *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 21]*

The function f is defined by $f(x) = \dfrac{2x-1}{x+2}$, with domain $D = \{x : -1 \le x \le 8\}$.

(a) Express $f(x)$ in the form $A + \dfrac{B}{x+2}$, where A and $B \in \mathbb{Z}$. *[2 marks]*

(b) Hence show that $f'(x) > 0$ on D. *[2 marks]*

(c) State the range of f. *[2 marks]*

(d) (i) Find an expression for $f^{-1}(x)$.

(ii) Sketch the graph of $y = f(x)$, showing the points of intersection with both axes.

(iii) On the same diagram, sketch the graph of $y = f^{-1}(x)$. *[8 marks]*

(e) (i) On a different diagram, sketch the graph of $y = f(|x|)$ where $x \in D$.

(ii) Find all solutions of the equation $f(|x|) = -\dfrac{1}{4}$. *[7 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 18]*

(a) (i) Express each of the complex numbers $z_1 = \sqrt{3} + \mathrm{i}$, $z_2 = -\sqrt{3} + \mathrm{i}$ and $z_3 = -2\mathrm{i}$ in modulus-argument form.

(ii) Hence show that the points in the complex plane representing z_1, z_2 and z_3 form the vertices of an equilateral triangle.

(iii) Show that $z_1^{3n} + z_2^{3n} = 2z_3^{3n}$ where $n \in \mathbb{N}$. *[9 marks]*

(b) (i) State the solutions of the equation $z^7 = 1$ for $z \in \mathbb{C}$, giving them in modulus-argument form.

(ii) If w is the solution to $z^7 = 1$ with least positive argument, determine the argument of $1 + w$. Express your answer in terms of π.

(iii) Show that $z^2 - 2z\cos\left(\frac{2\pi}{7}\right) + 1$ is a factor of the polynomial $z^7 - 1$. State the two other quadratic factors with real coefficients. *[9 marks]*

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